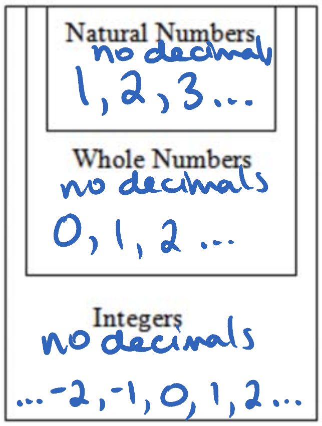
6.1 Graphing Linear Inequalities

Graphing Linear Inequalities in Two Variables [6.1]

Let's start with a little Review. What can you tell me about:



Real #'s $\mathbb{R}$ – all #'s, decimals, fractions, etc

Integers I

Whole #'s W

Natural #'s N

Real #'s are the only group here that have decimals.

A mathematical inequality must contain one of the following symbols:

less than or equal to

$<$ $\leq$ $>$ $\geq$ $\neq$

less than greater than

The following are examples of linear inequalities in a single variable :

$4x - 1 > 7$ $\qquad$ $1 - 2a \leq 5$ $\qquad$ etc.

The solution to a single variable inequality can be shown on a number line.

0

Example 1: Graph the solution set for $-3x + 4y \leq 12$

+3x +3x

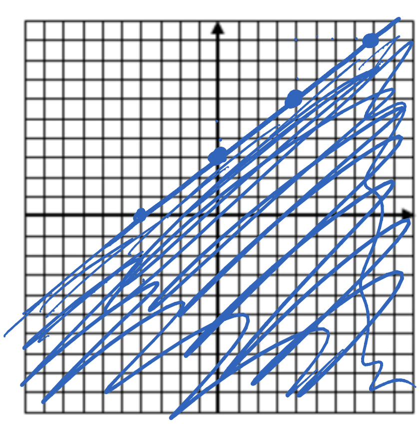
less than or equal to → solid line

① solve for y.

$$\frac{4y}{4} \leq \frac{12}{4} + \frac{3x}{4}$$

$$y \leq \frac{3}{4}x + 3$$

slope $= \frac{3}{4}$ y-intercept



② plot the y-intercept (0,3)

③ use the slope to get 2 more pts

$\frac{3}{4} = \frac{\text{rise}}{\text{run}}$

$\frac{-3}{-4} = \frac{3}{4}$

test pt (0,0) into original eq

$-3(0) + 4(0) \overset{?}{\leq} 12$

-4 4 $-3(0)+4(0)\leq 12$

$0 \leq 12$ True

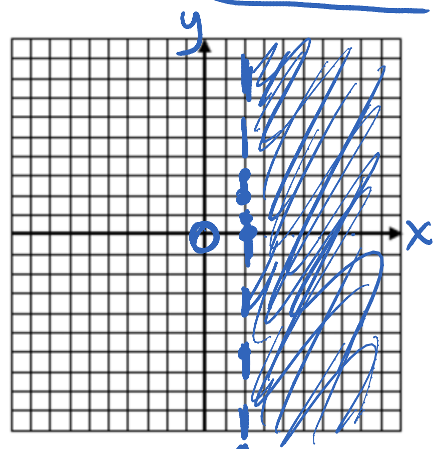
So shade the (0,0) side of line

$\frac{3}{4} = \frac{rise}{run}$

Example 2: Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in R, y \in R\}$ — we shade, rather than stipple

line will be dashed



$x = 2$

$x > 2$

Test pt $(0,0)$

$0 \overset{?}{>} 2$ not true so shade other side

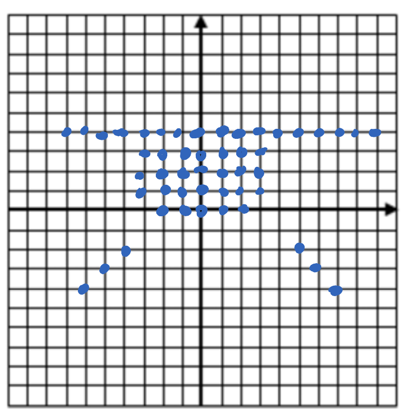
b) $\{(x,y) \mid -3y + 6 \geq -6, x \in I, y \in I\}$

~~solid~~ stippled line

stipple the solution area, rather than shade.

no decimals in solution

$y=4$



$-3y + 6 \geq -6$

$-3y \geq -6 - 6$

$-3y \geq -12$

$y \leq \frac{-12}{-3}$

$\underbrace{y \leq 4}_{\text{horizontal line}}$

when we divide or multiply by a negative we must FLIP the inequality

Test pt $(0,0)$

$0 \overset{?}{\leq} 4$ True so stipple below the line

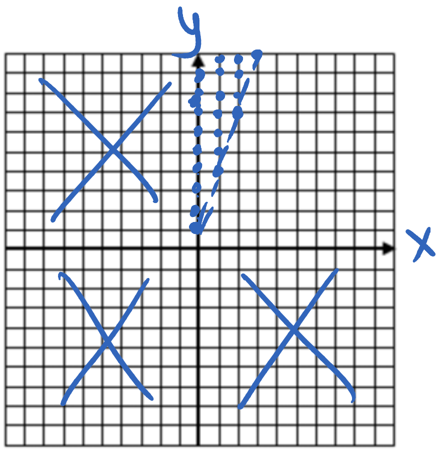
Example 3:

Oliver and Connor are competing in a spelling quiz. Connor (y) gets a point for every word he spells correctly. Oliver (x) is younger than Connor, so he gets 3 points for every word he spells correctly, plus one bonus point. What combinations of correctly spelled words ~~for Oliver and for Connor scoring more points than Oliver~~? Choose two combinations that make sense and explain your choices.

Will allow Connor to win?

\# of words Connor $= y$ \# of words Oliver $= x$

$3x+1$



Connor to win

$y > 3x+1$

dashed

$x, y \in W$

spelling

Slope $= \frac{3}{1} = \frac{\text{rise}}{\text{run}}$

y-int $= 1$

test pt $(0,0)$

$0 \overset{?}{>} 3(0)+1$

$0 \overset{?}{>} 1$ Not true so shade other side from $(0,0)$

2 combinations that make sense: (any of the dots)

Connor scores 2, Oliver 0

OR

Connor scores 6, Oliver 1

6.1 Summary: Type of line/shading for Inequality Graph

Solid line: $\leq$ or $\geq$ and $x, y \in \mathbb{R}$ (continuous)

Stippled line: $\leq$ or $\geq$ and (discrete) $x, y \in W$
$\in I$
$\in N$

Dashed line: $<$ or $>$ x, y can be $\mathbb{R}, W, I, N$

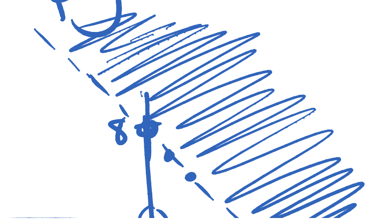
Full shading: when continuous $x, y \in \mathbb{R}$

Stippled: when discrete $x, y \in I$
$\in W$
$\in N$

Check for which side to stipple or shade by using a test point (0,0 if it isn't on the line)

Practice pg 303 # 4, 5ace, 6ace, 7, 10, 12

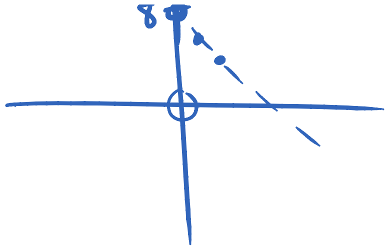
5a



$y > -2x + 8$
slope
y int

slope $= \frac{-2}{1} = \frac{2}{-1}$
- rise

5a



↑ slope ↑ y int

dashed line - - - -

= rise / run

assume $x, y \in R$

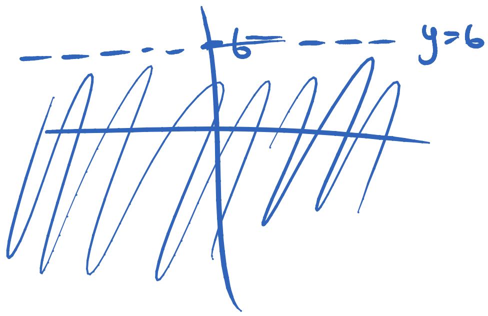
shading

Test pt (0,0)

$0 \overset{?}{>} -2(0)+8$

$0 \overset{?}{>} 8$ not true so shade side without test pt

c



$y < 6$ $x, y \in R$ shading

↑ dashed line

e $10x - 12 < -y$

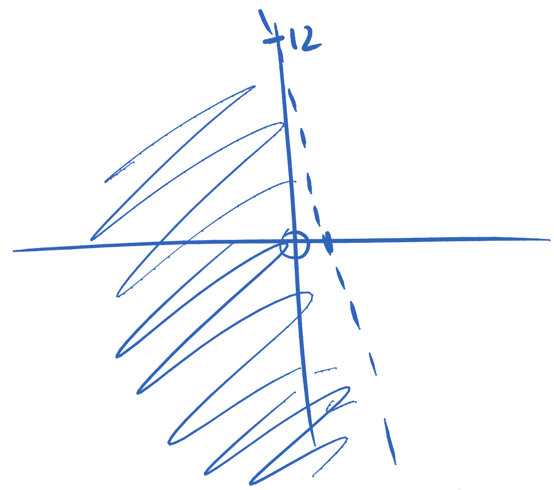
↑ dashed line

x	y
0	12
$\frac{6}{5}$	0

$10x - 12 < 0$

$10x < +12$

$x < \frac{6}{5}$



Test pt (0,0)

$10(0) - 12 \overset{?}{<} 0$

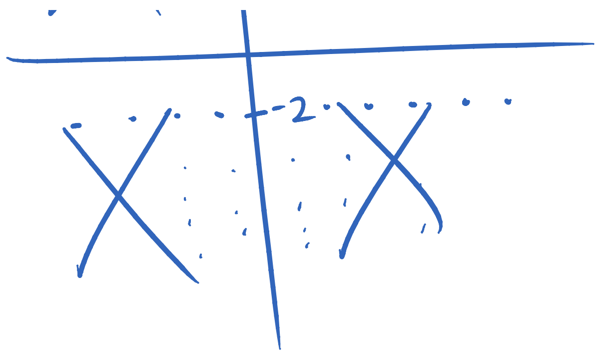
$-12 \overset{?}{<} 0$ yes

6a

$x, y \in W$ → stippling

0, 1, 2, ...

no negatives



no negatives
no decimals

$$2x - y \geq 5y + 2x + 12$$

not dashed because is $\geq$ only we stipple the line too

$$\cancel{2x} - 2x - 12 \geq 5y + y$$

$$\frac{-12}{6} \geq \frac{6y}{6}$$

$y = (\text{slope})x + \text{yint}$

$y = mx + b$

$$-2 \geq y \longrightarrow y \leq -2$$

since only whole #'s and whole #'s aren't negative, there is no solution.

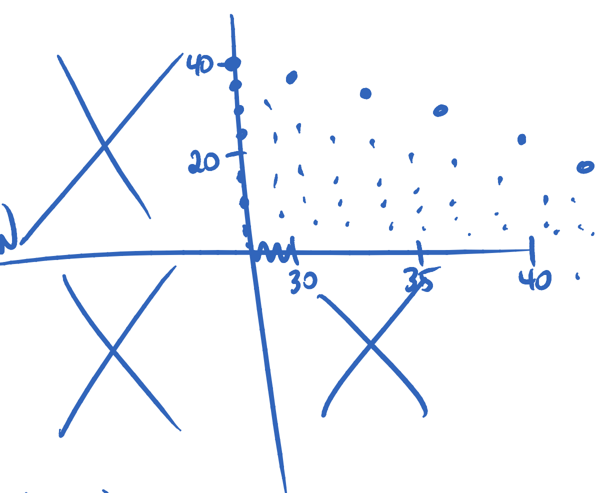
12) x = # of rect. tables $\quad x, y \in W$

y = # of circular tables

a) $12x + 8y \leq 660$

stippled since $x, y \in W$

x	y
30	37.5
33	33
40	22.5



test pt (0,0)

$12(0) + 8(0) \overset{?}{\leq} 660$? True

So stipple on (0,0) side

6.2 Exploring Graphs of Systems

Exploring Graphs of Systems of Linear Inequalities [6.2]

→ 2 equations

The "solution" is where they overlap.

Example 1:

Graph this system of linear inequalities. Justify your representation of the solution set.

solid stippled

$$\{(x, y) | x + y \leq 5, x \in I, y \in I\}$$

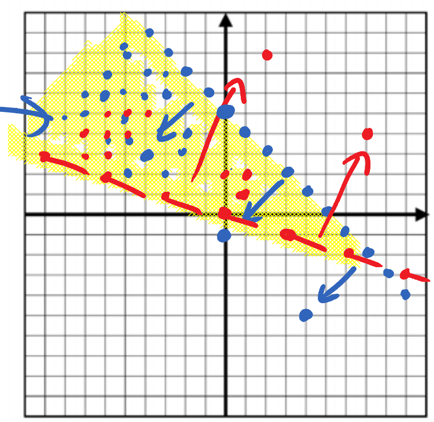
$$\{(x, y) | x + 3y > 0, x \in I, y \in I\}$$

dashed stipled

① $x + y \leq 5$

② $x + 3y > 0$

Stippled



① $y \leq -x + 5$

slope $= \frac{-1}{1}$ y int $= 5$

test pt $(0,0)$

$x + y \overset{?}{\leq} 5$

$0 + 0 \overset{?}{\leq} 5$ yes

② $x + 3y > 0$

$3y > -x$

$y > -\frac{x}{3} + 0$

slope $= -\frac{1}{3} = \frac{rise}{run}$ y int $= 0$

$= \frac{1}{-3}$

test point $(1,1)$

$1 + 3(1) \overset{?}{>} 0$

$4 \overset{?}{>} 0$ yes

stipple the side that has the test point

Where the 2 solution areas (stippled) overlap is the solution of the system of equations.

6.2 Practice Worksheet

Wednesday, November 20, 2013
3:15 PM

Kuta Software - Infinite Algebra 1 6.2 Practice

Name________________

Solving Systems of Inequalities

Date________________ Period____

Sketch the solution to each system of inequalities.

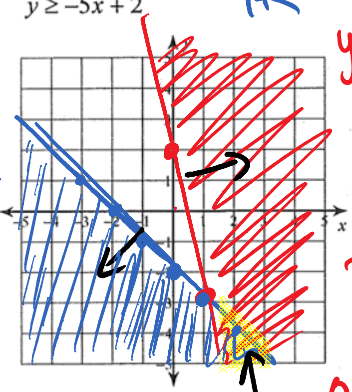
Assume $\boxed{x, y \in \mathbb{R}}$ unless told otherwise

1) $y \le -x - 2$
$y \ge -5x + 2$

$\mathbb{R}$

$y \le -x - 2$
slope $= \frac{-1}{1} = \frac{1}{-1}$, y int -2

Test pt $(0,0)$
$0 \overset{?}{\le} -0 - 2$
$0 \overset{?}{\le} -2$ False
$\therefore$ shade other side

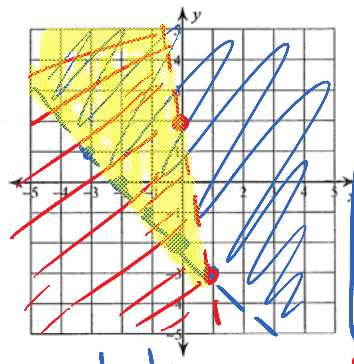


overlapping area is the solution

$y \ge -5x + 2$
slope $= \frac{-5}{1}$, y-int

Test pt $(0,0)$
$0 \overset{?}{\ge} -5(0) + 2$
$0 \overset{?}{\ge} 2$ False
$\therefore$ shade other side

2) $y > -5x - 2$ (slope, y int)
$y < -5x + 2$



test p $(0,0)$
$0 \overset{?}{>} -0 - 2$
$0 \overset{?}{>} -2$ true

Slope $\left(\frac{-5}{1}\right) = \frac{\text{rise}}{\text{run}}$
down 5, right 1
slope $\frac{5}{-1}$
up 5, left 1

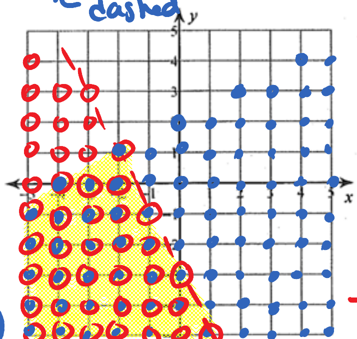
test pt $(0,0)$
$0 \overset{?}{<} -5(0) + 2$
$0 \overset{?}{<} 2$ true

3) $y \le \frac{1}{2}x + 2$ (stippled)
$y < -2x - 3$ (dashed)

$\boxed{x, y \in \mathbb{I}}$

$y \le \frac{1}{2}x + 2$
slope, y int
- stippled line
- test pt $(0,0)$

$0 \overset{?}{\le} \frac{1}{2}(0) + 2$
$0 \overset{?}{\le} 2$ True
$\therefore$ stipple the $(0,0)$ side.



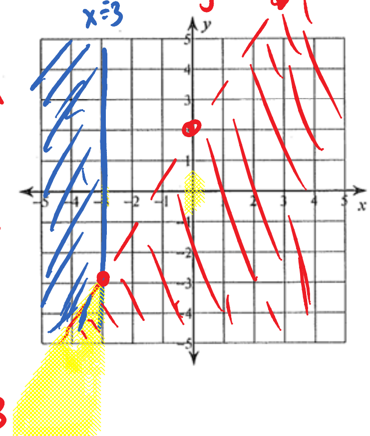
Solution area (overlapping)

$y < -2x - 3$
slope $-\frac{2}{1} = \frac{2}{-1}$, y int
- dashed line
- test pt $(0,0)$

$0 \overset{?}{<} -2(0) - 3$
$0 \overset{?}{<} -3$ False
$\therefore$ stipple other side

4) $x \le -3$ (slope)
$y < \frac{5}{3}x + 2$ (y-int)

$x = 3$

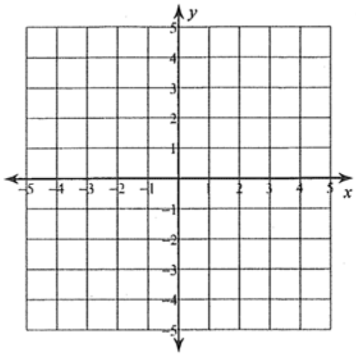


Slope $= \frac{-5}{-3} = \frac{5}{3}$
down 5, left 3; up 5, rt 3

$0 \overset{?}{<} 2$ True

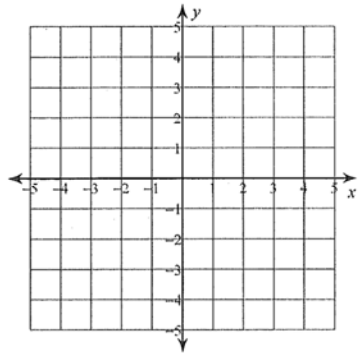
5) $y \le -\frac{5}{2}x - 2$

$y < -\frac{1}{2}x + 2$



6) $y \ge \frac{2}{3}x + 3$

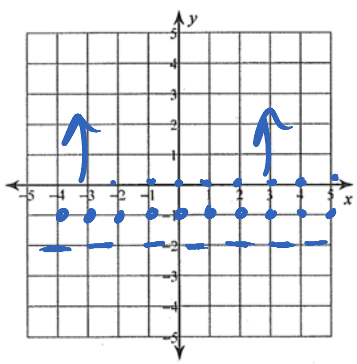
$y > -\frac{4}{3}x - 3$



7) ~~$4x + y < 2$~~

$y > -2$

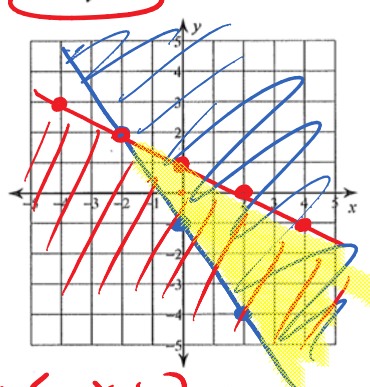
$x, y \in I$



8) $3x + 2y \ge -2$

$x + 2y \le 2$

solid lines
shaded



$2y \ge -2 - 3x$

$2y \ge -3x - 2$

$y \ge \frac{-3}{2}x - 1$

slope $= \frac{3}{-2}$ y-int

Test pt $(0,0)$

$3(0) + 2(0) \overset{?}{\ge} -2$

$0 \overset{?}{\ge} -2$ True

∴ shade $(0,0)$ side

$2y \le -x + 2$

$y \le -\frac{1}{2}x + 1$

slope yint

$-\frac{1}{2}$ or $\frac{1}{-2}$

test pt $(0,0)$

$0 + 2(0) \overset{?}{\le} 2$

$0 \le 2$ True

so shade the test pt side

Extra Practice
pg 307 # ~~1~~, 2

pg 307

test pt side

pg 307

(2) $-x+2y \geq -4$ ER

$y \geq x$

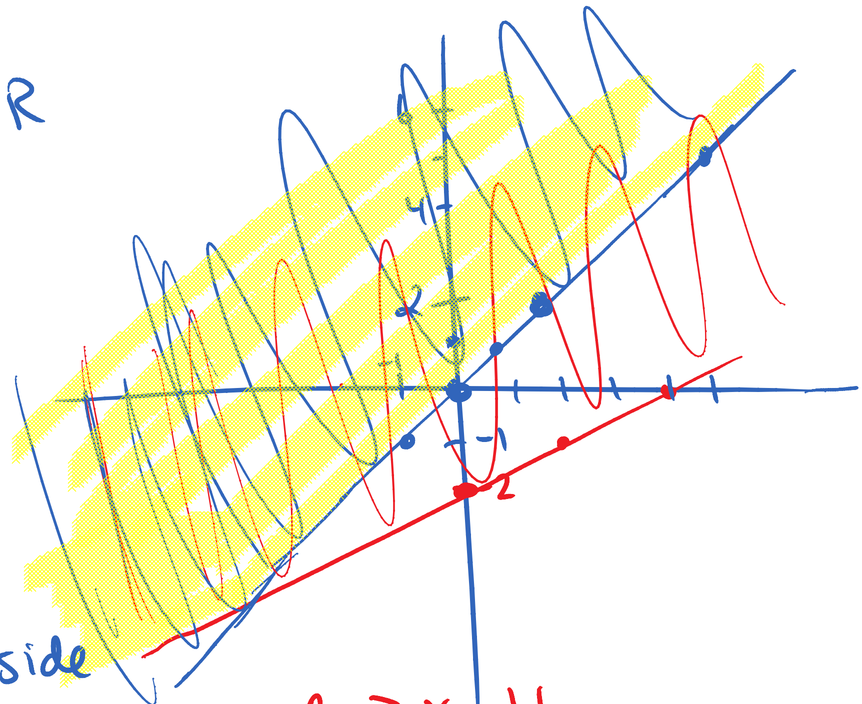
$y=x$

x	y
0	0
2	2
5	5
-1	-1

Test pt (0,1)

$1 \overset{?}{\geq} 0$ True

shade the (0,1) side



test point (0,0)

$-0+2(0) \overset{?}{\geq} -4$

$0 \overset{?}{\geq} -4$

$2y \geq x - 4$

$y \geq \frac{1}{2}x - 2$

slope y-int

Graphing Systems of Linear Inequalities

Tuesday, November 26, 2013
12:12 PM

Line

— solid if $\leq$ or $\geq$ **and** $\in \mathbb{R}$ ← include decimals

••• stippled if $\leq$ or $\geq$ and $\in I, W,$ or N

--- dashed if $<$ or $>$ and any number system

Solution Area

shaded if $\in \mathbb{R}$

stippled if $\in I, W,$ or N

Graphing

$y = mx + b$ (m = slope, b = y-intercept)

1. Graph line (deciding if solid, stippled, or dashed)
2. Choose test point, not on the line, plug into original equation. If true then shade/stipple the side including test point. If false then shade/stipple other side.
3. Do the same for the other equation
4. Where the shaded/stippled areas overlap is the solution of the system of inequalities.

6.3 Graphing to Solve Systems of Linear Inequalities

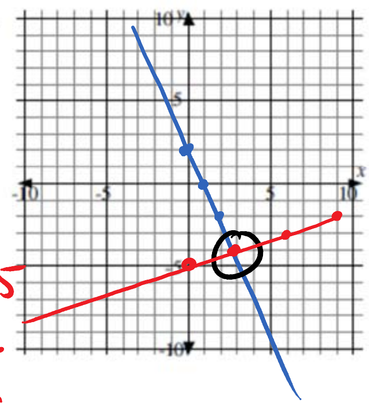
Graphing to Solve Systems of Linear Inequalities [6.3]

A quick review:

Consider the system of equations $2x + y = 2$, $x - 3y = 15$.

a) Solve the system of equations graphically by:

- writing both equations in slope y-intercept form
- making a table of values and plotting the points.



$y = -2x + 2$

slope $= \frac{-2}{1} = \frac{2}{-1}$

y intercept

$-3y = -x + 15$

$y = \frac{1}{3}x - 5$

slope

y-int

Solution is where the 2 lines cross: (3, −4)

Example 1:

Graph the solution set for the following system of inequalities. State two possible solutions from the set. Check your work.

dashed

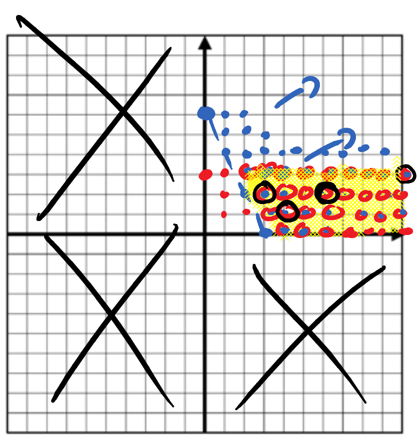
$\{(x, y) | 2x + y > 6, x \in W, \ y \in W\}$

$\{(x, y) | y \leq 3, x \in W, y \in W\}$

stippled line since W #'s.

W: {0, 1, 2...}

stippled



$y > -2x + 6$

slope $= \frac{-2}{1}$

y int

test pt (0,0)

$2(0) + 0 \overset{?}{>} 6$ False

∴ stipple other side from test pt

$y \leq 3$

Possible Solutions

(6,2)

(3,2)

(4,1)

Example 2:

Graph the system and determine a solution.

dashed line shading

$\{(x, y) | y - x < 2, x \in R, \ y \in R\}$ → $y < x + 2$

$\{(x, y) | x + y \geq 0, x \in R, \ y \in R\}$

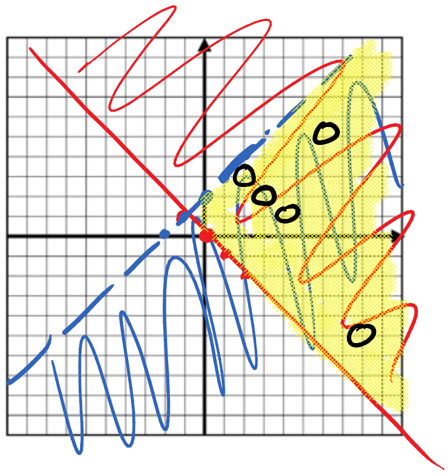
solid line

slope $\frac{1}{1}$ y-int

test pt (0,0)

$0 - 0 \overset{?}{<} 2$ True

so shade (0,0) side



$y \geq -x + 0$

$\frac{-1}{1}$ or $\frac{1}{-1}$

test pt (1,1)

$1 + 1 \overset{?}{\geq} 0$ True

shade (1,1) side

Example 3:

To raise funds to buy new instruments, the band committee has 500 T-shirts to sell. The T-shirts come in red or blue. Based on sales of the same T-shirts at a fundraiser five years ago, the committee expects to sell at least twice as many blue T-shirts as red T-shirts.

a.) Define the variables and restrictions. Write a system of linear inequalities that models the situation.

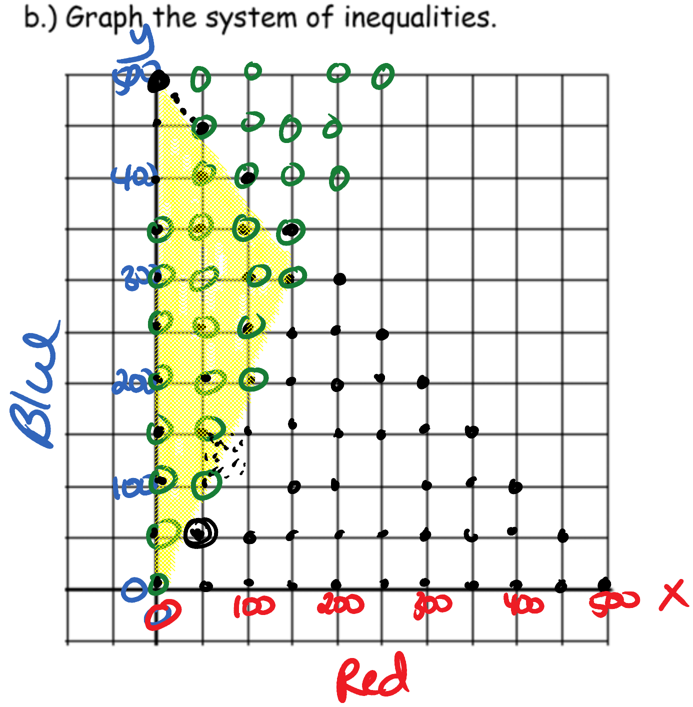
\# of Red T-shirts : x

\# of Blue T-shirts : y

$x, y \in W$ → stippled

can't sell neg or portions of a shirt.

b.) Graph the system of inequalities.



$x + y \leq 500$

$y \geq 2x$

$y \leq -x + 500$

slope: $\frac{-1}{1} = \frac{-100}{100}$

y-int.

test pt (0,0)

$0 + 0 \overset{?}{\leq} 500$

$y \geq 2x + 0$

slope $\frac{2}{1} = \frac{100}{50}$

y-int

Test pt (50,50)

$50 \overset{?}{\geq} 2(50)$ False

c.) Suggest a combination of T-shirt sales that could be made.

Possible #'s

50 Red, 200 Blue

150 Red, 300 Blue

150 Red, 350 Blue

6.3 Homework

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if you need more system graphing practice then do # 4,5

Every should do # 6, 8, 10

6 a) $x =$ # of egg $y =$ # of ham

$x + y \le 450$

$y \ge 2x + 0$

y int

$y \le -x + 450$

y-int

test pt (0,0)

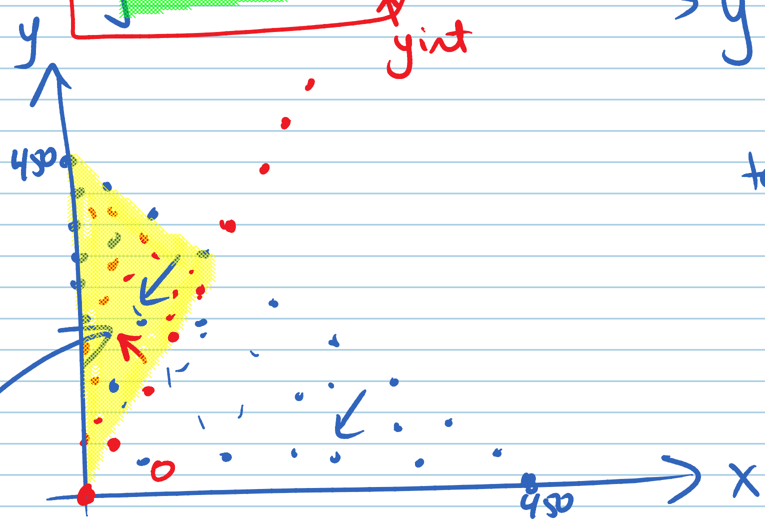
$0 \overset{?}{\le} 450$ yes

∴ stipple side of line with test point

b) $x, y \in W$

stippled

c)



test pt (2,1)

$1 \overset{?}{\ge} 2(2)$

$1 \overset{?}{\ge} 4$ No

∴ ~~shade~~ stipple other side

d) any point (x,y) from the solution area

$x =$ egg $y =$ ham

8 a) $x =$ # of school friends

$y =$ # of rugby friends

~~$3x = y$~~

# of rugby friends

y = # of rugby friends

# of rugby friends

$x + y \leq 500$

$x \geq 3y$

b) Restrictions

$x, y \in W$

test pt (0,0)

$0 \overset{?}{\leq} 500$ yes

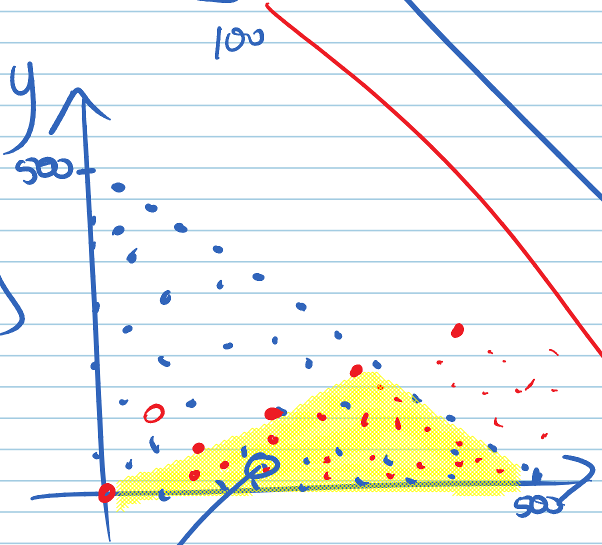
$y \leq \frac{1}{3}x$

y-int=0

test pt (10,10)

$10 \overset{?}{\geq} 3(10)$ No

c)



(200, 25)
school rugby

⑩ max 36 songs

x = # of old songs

y = # of young songs

$x + y \leq 36$

$y \geq 2x$ ☆

old

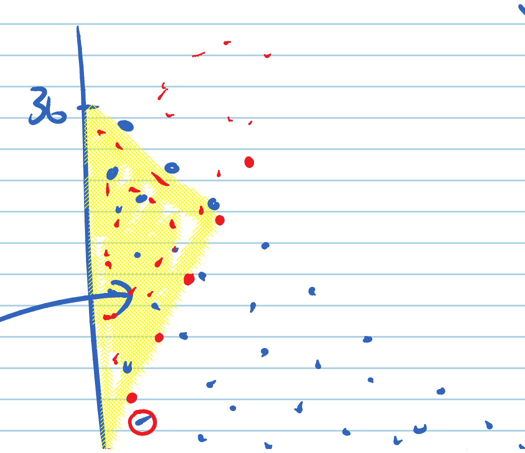
5

$x, y \in W$

test (1,1)

$1 \overset{?}{\geq} 2(1)$ No

∴ stipple other side



(2, 18)
young

(α, 10)
old young

Practice pg 323

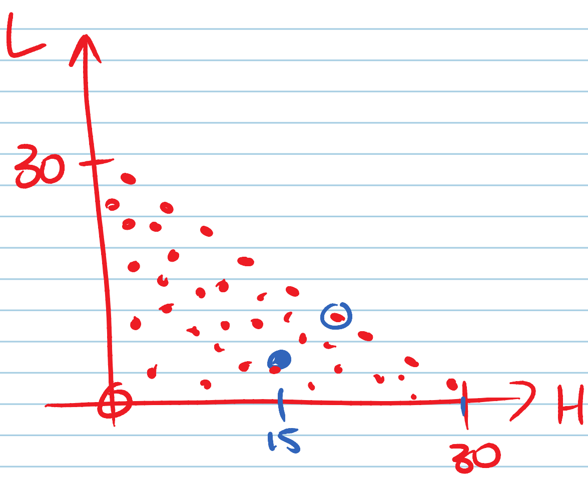
#2 – 5, 6 (set up only, no graph), 7a

③ a) H = # of hours Horst works ✓
L = " " " Lev "

$H + L \leq 30$ ✓ $H, L \in W$

stippled lines + areas

b)



test pt (0,0)

$0 + 0 \overset{?}{\leq} 30$

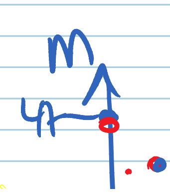
True

so ~~shade~~ stipple (0,0) side.

Horst	15 hrs	30 hrs	0	18
Lev	5 hrs	0	30	12

7a

P = # hrs Pali works $P, M \in W$
M = " " Meg " stipple



$\overset{0}{18P} + \overset{0}{10M} \overset{?}{\leq} 470$ True

$18P + 10M \leq 470$

$\overset{0}{18P} + \overset{0}{10m} \le 470$ true

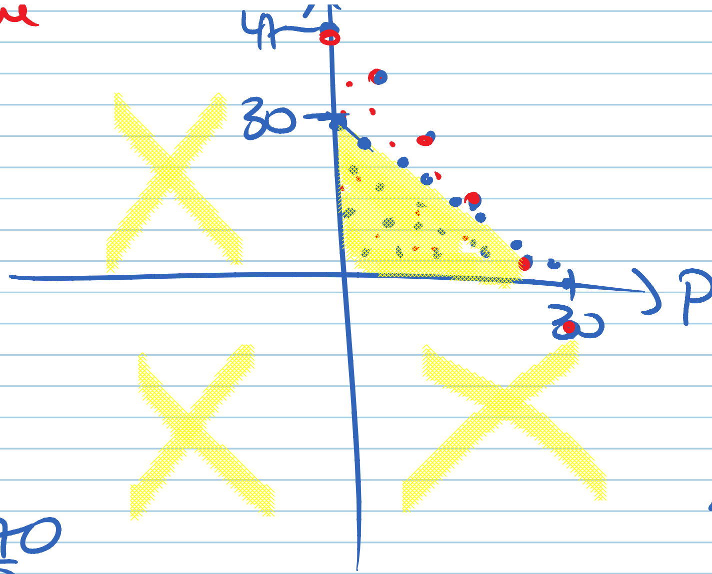
$$18P + 10m \le 470$$

$$P + m \le 30$$

solve for m(y)

$$\frac{10m}{10} \le -\frac{18P}{10} + \frac{470}{10}$$

$$m \le -\frac{9}{5}p + 47$$



slope $= \frac{\text{rise}}{\text{run}} = \frac{-9}{5}$